Topology Control in Wireless Networks

4/24/06

Topology control

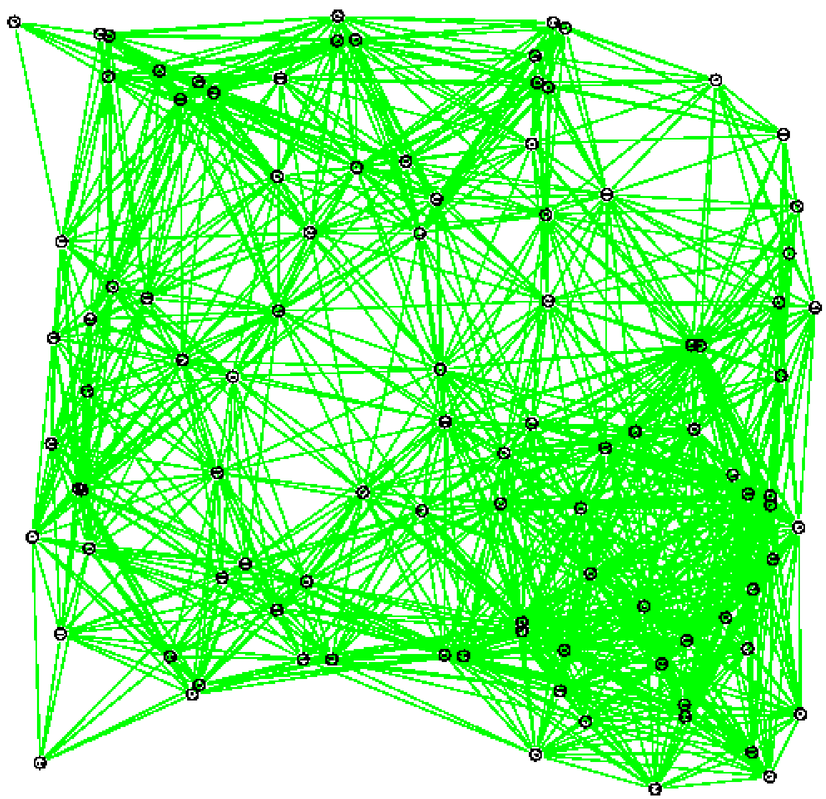
- Choose the transmission power of the nodes so as to satisfy some properties
  - Connectivity
  - Minimize power consumption, etc.
- Last class
  - Percolation: focuses on connectivity.
  - Assume all the nodes use the same transmission range.
  - The nodes are uniformly randomly deployed.

Topology control

- Nodes may choose different transmission range.
  - The node density is not uniform.
- Minimize the total power consumption.
  - The power consumption is proportional to d^{α}, where d is the transmission range, α is a constant within 2 and 5.

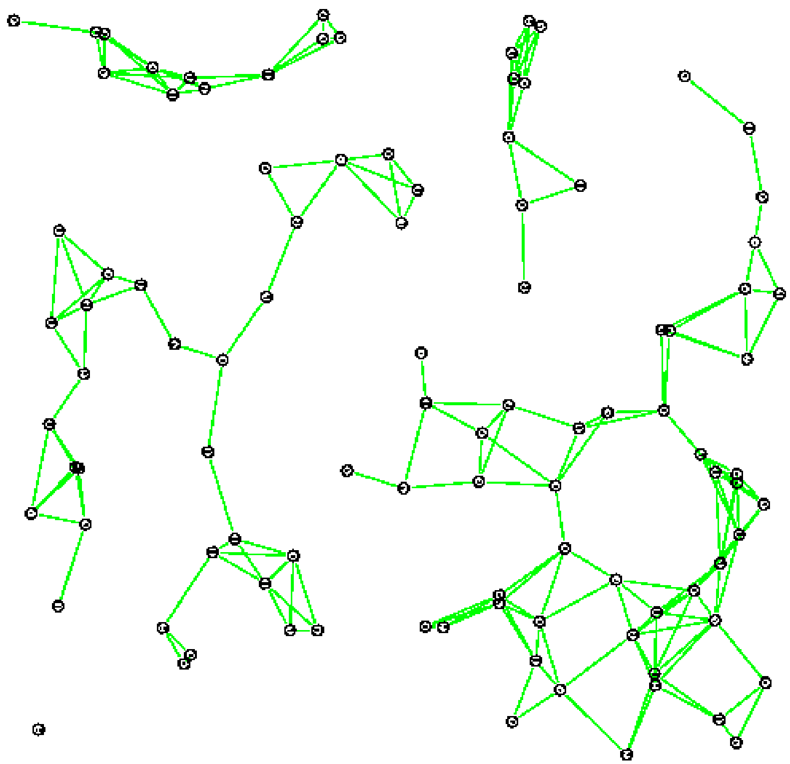
Motivation for Topology Control

- Example of No Topology Control with maximum transmission radius *R* (maximum connected node set)



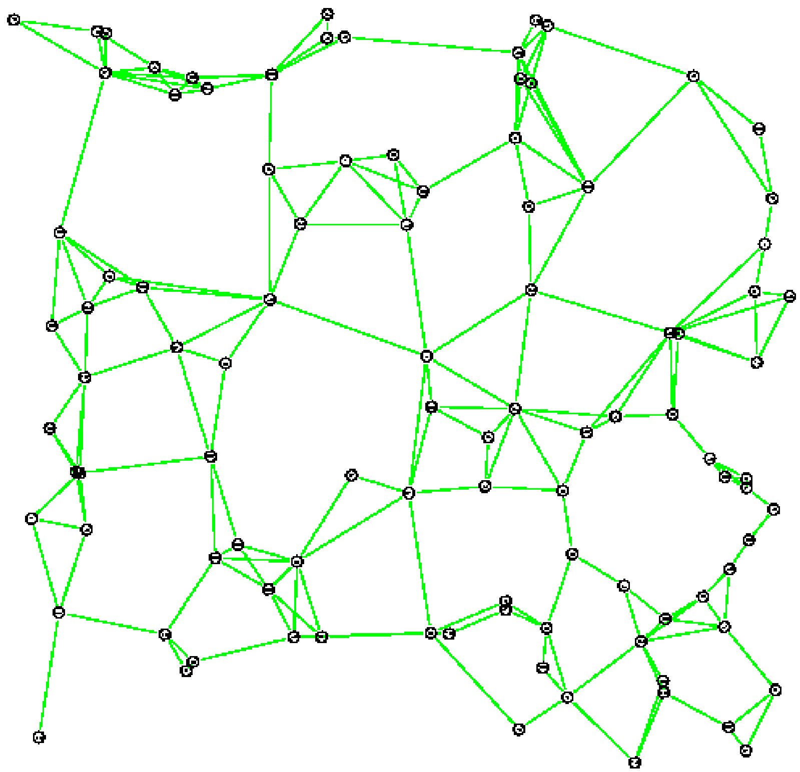
- High energy consumption
- High interference
- Low throughput

- Example of No Topology Control with smaller transmission radius



- Network may partition

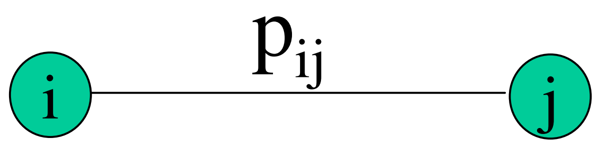
- Example of Topology Control



- Global connectivity
- Low energy consumption
- Low interference
- High throughput

Power Optimal Routing

- Label a link from node i to node j with a cost p_{ij} that equals to the minimum transmission power that i needs in order to reach j



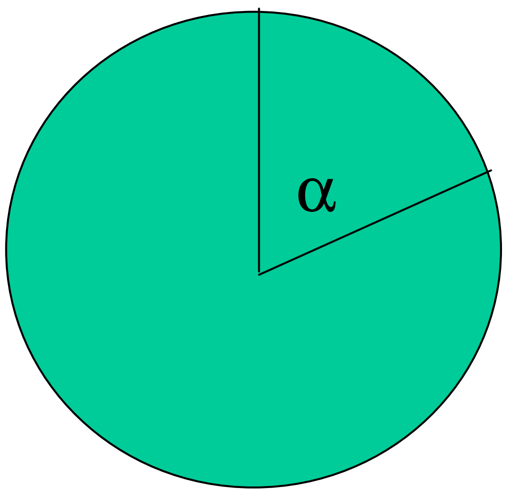
  - if i cannot reach j with its maximum power, the cost is ∞
  - assume symmetric channels in our discussions, i.e., $p_{ij} = p_{ji}$
- Then run a shortest path algorithm over this graph for each source-destination pair

Topology Control with Transmission Power Management

- Overhead of running a complete power optimal routing protocol
  - storage overhead
    - for each destination: the next hop neighbor and the corresponding transmission power
  - discovery overhead
    - each node needs to discover the minimum power from it to reach all other nodes
- Topology control with transmission power management
  - maintain only a subset of the links
  - the graph with the subset of links should be connected and have close to optimal performance

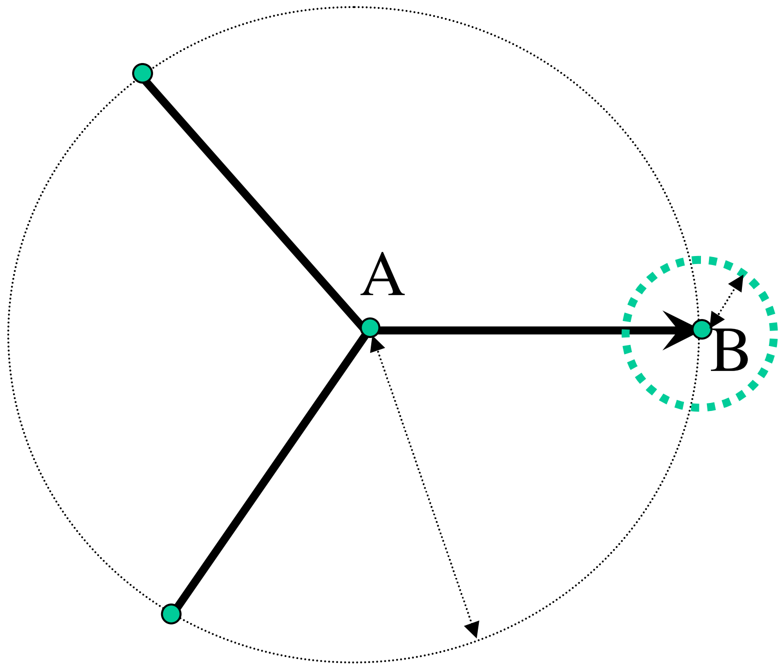
Cone-Based Topology Control (CBTC)

- Instead of determining the minimum transmission power to all other nodes, node A increases its transmission power to have at least one neighbor in any α cone



- Implementation:
  - sends HELLO message
  - needs to be able to determine angle of arrival (AoA)

Properties of CBTC: Asymmetric Links



A needs B to have a neighbor

B has more neighbors and thus uses a lower power

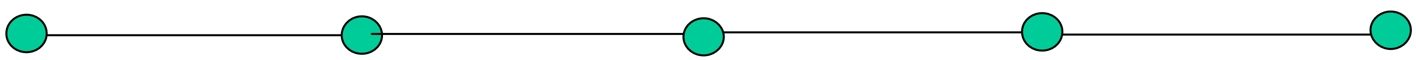
Symmetric Closure of G^{α}

- Algorithm
  - first runs the cone-based algorithm
  - then do adjustment
    - if a node A is using another node B as its neighbor, A sends B its power level
    - B raises its power level to that of A if A's power level is higher
  - the final graph $G^{\alpha,c}$:
    - each node has a power level
    - there is link from a node to all other nodes reachable using its power level
    - it is a super graph of G^{α}
- Is the final graph $G^{\alpha,c}$ symmetric, i.e., if $A \rightarrow B$, then $B \rightarrow A$?
- If $A \rightarrow B$ in G^{α}, is $B \rightarrow A$ in $G^{\alpha,c}$?

Properties of CBTC

- A node may not be able to find at least one neighbor in any α cone

example: if $\alpha \leqslant 120^{o}$



- If a node increases its power to the maximum but there still exists an α cone with no node, the algorithm fails

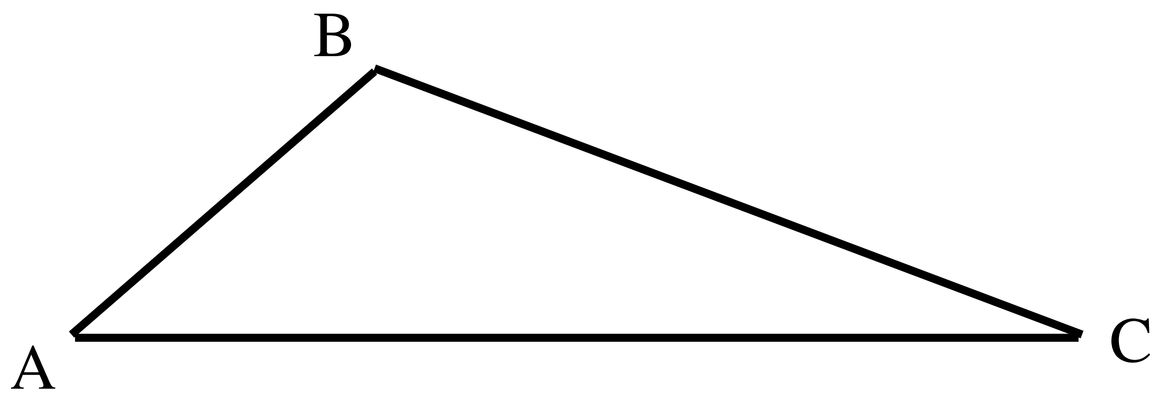
From Local Property to Global Property

- Assume the graph G^M when every node is using its maximum power is connected

- Assume the algorithm finishes successfully, i.e., all nodes finish; call the graph G^α

- Result: if $\alpha \leqslant 2\pi/3$ (= 120°), then G^α is connected

Two Facts about Triangle



- If angle $A \geq$ angle $C \Leftrightarrow BC \geq AB$
  - in words, longer edge length corresponds to larger angle

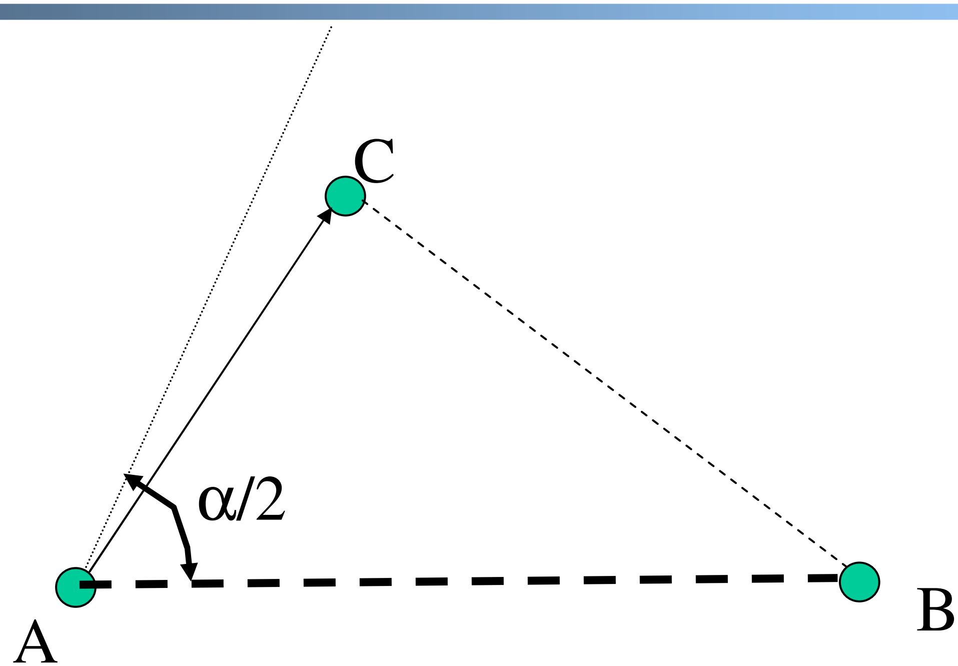
- $AB^2 + BC^2 - 2AB \cdot BC \cos B = AC^2$

Connectivity of CBTC

- Proof by contradiction: assume the network G^{α} is not connected

- Choose nodes A and B such that
  - A cannot reach B in G^{α}
  - their distance is the minimum among all pairs (C, D) where C cannot reach D in G^{α}

Connectivity



- Relationship among edge lengths
  - AC < AB (since A can reach C but not B)
  - AB ⩽ CB
    - since there is no CB edge; otherwise A is connected to B
    - since AB has the minimum distance among all disconnected pairs, AB ⩽CB
  - thus AC < AB ⩽ CB
- However, the angle at A (CAB) is ⩽ 60°, thus the sum of degrees is less than 180°; contradiction

Competitive Analysis

- Objective: show that the power consumption in the sparser network is not too bad compared with that of power optimal routing where you have all the links

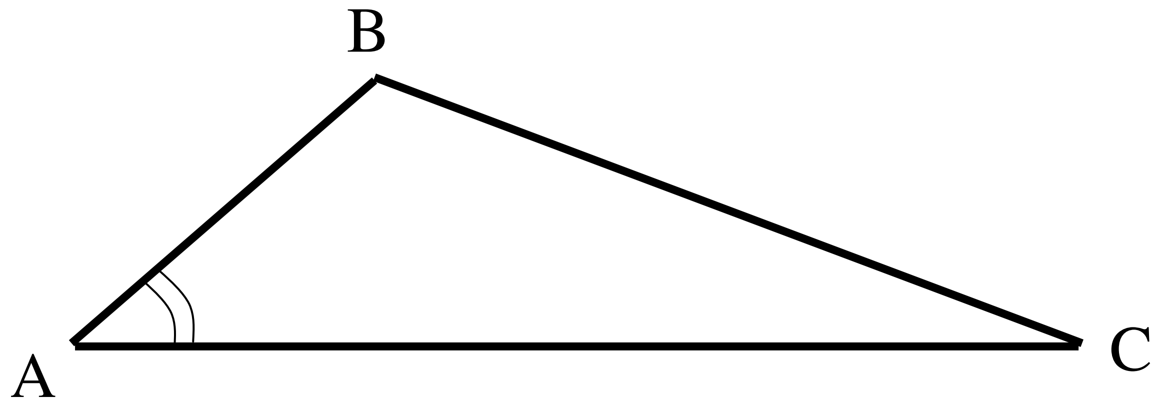
- Competitive ratio:

$$\frac{\text{the energy cost using the cone-based algorithm}}{\text{the energy cost using power optimal routing}}$$

Competitive Analysis

- Power model:
  - power level $p(d)$ needed to reach distance d: $cd^x \leqslant p(d) \leqslant z\, cd^x$, where $x \geq 2$, c is a constant, and z is a constant ≥ 1.
- Result: Competitive ratio of the cone-based algorithm is $z(1+2\sin(\alpha/2))$, for $\alpha \leqslant \pi/2$.

Triangle: $AB \leqslant BC$, $A \leqslant 45°$



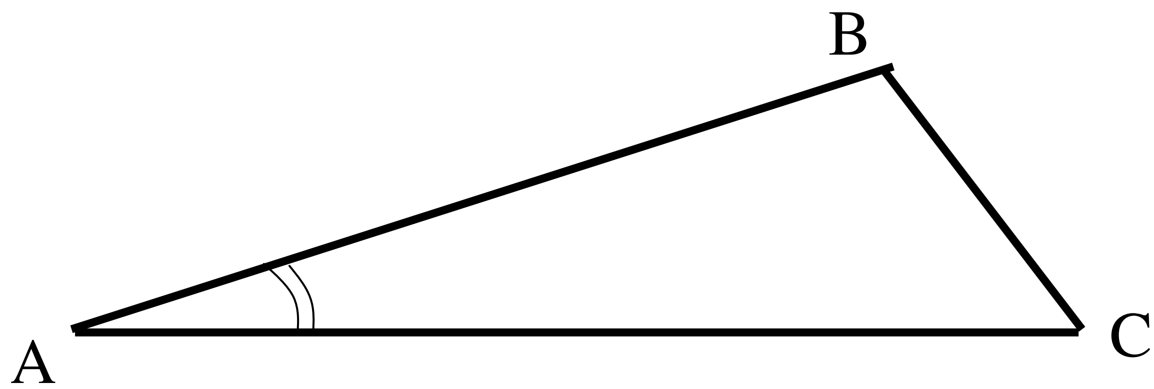
$$AB \le BC; A \le 45^\circ \Rightarrow C \le 45^\circ \Rightarrow \begin{cases} B \ge 90^\circ \\ AC \ge AB \\ AC \ge BC \end{cases}$$

$$AB^2 + BC^2 - 2AB \cdot BC \cos B = AC^2$$

$$\Rightarrow AB^2 + BC^2 \le AC^2$$

$$\Rightarrow AB^x + BC^x \le AC^{x-2}(AB^2 + BC^2) \le AC^x$$

Triangle: AC ≥ AB ≥ BC, A ⩽ 45°



$$AC \geq AB \geq BC; A \leq 45^\circ$$

$$A + B + C = 180^\circ \Rightarrow A + 2B \geq 180^\circ \Rightarrow B \geq 90^\circ - A/2$$

$$AB^2 + BC^2 - 2AB \cdot BC \cos B = AC^2$$

$$\Rightarrow AB^2 + BC^2 \leq AC^2 + 2AC^2 \cos(B)$$

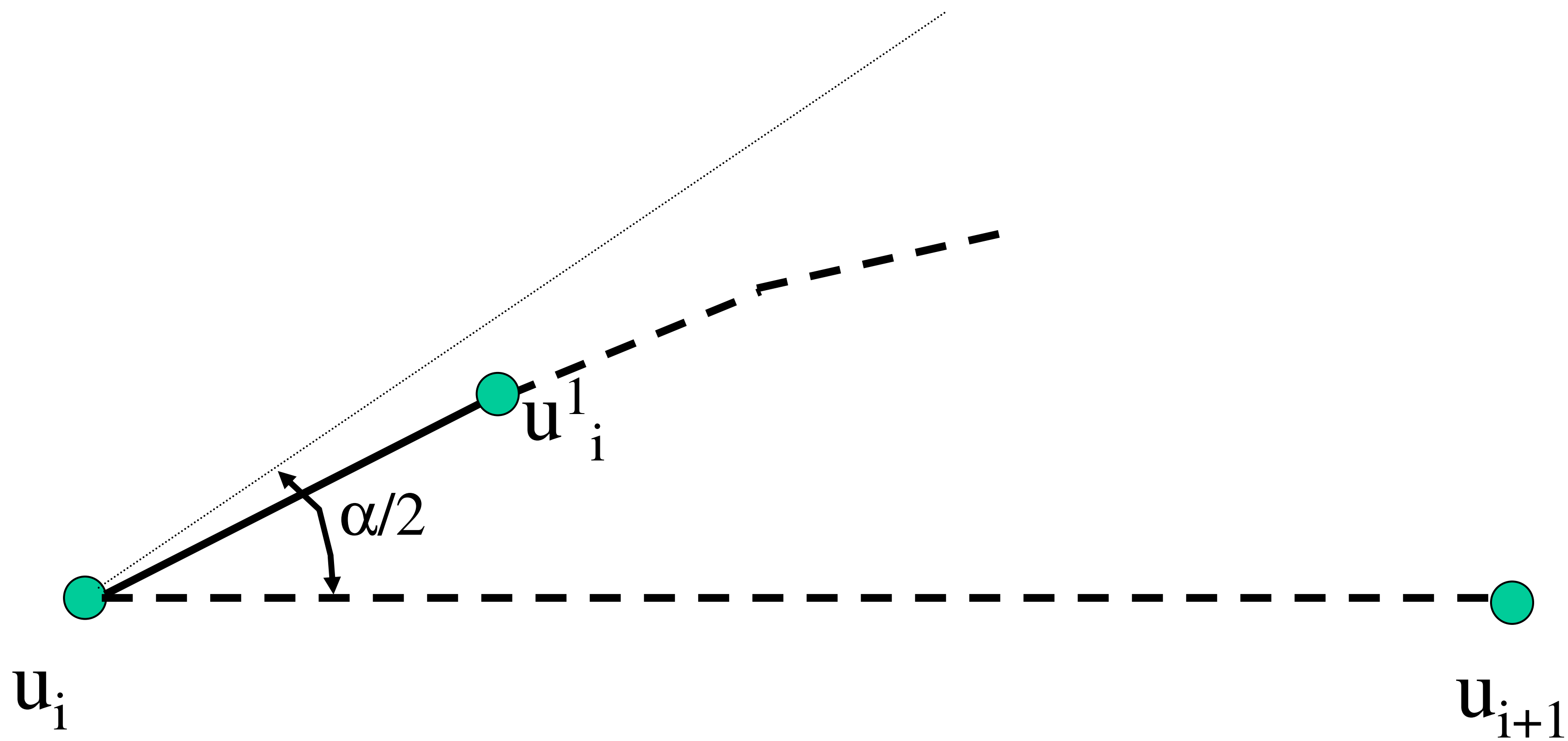
$$\Rightarrow AB^2 + BC^2 \leq AC^2(1 + 2\cos(90 - A/2)) = AC^2(1 + \sin(A/2))$$

$$\Rightarrow AB^x + BC^x \leq AC^{x-2}(AB^2 + BC^2) \leq AC^x(1 + \sin(A/2))$$

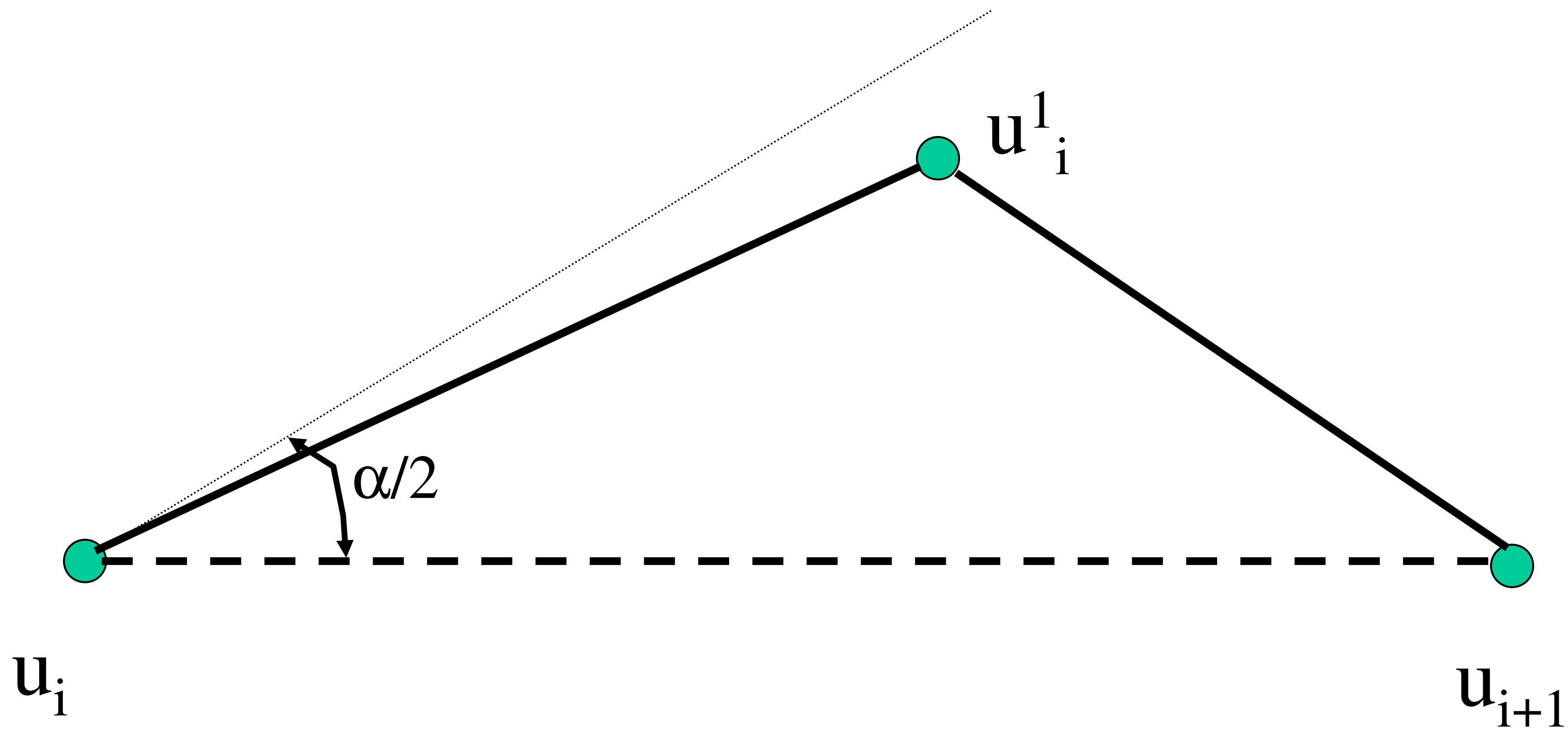
Deriving Competitive Ratio

- The power optimal path of a source destination pair uses a sequence of links in G^M
- Assume a link from u_i to u_{i+1} on the path is not in $G^{\alpha,C}$
- We can find a path to simulate the link u_i to u_{i+1} in $G^{\alpha,C}$ without paying too high cost

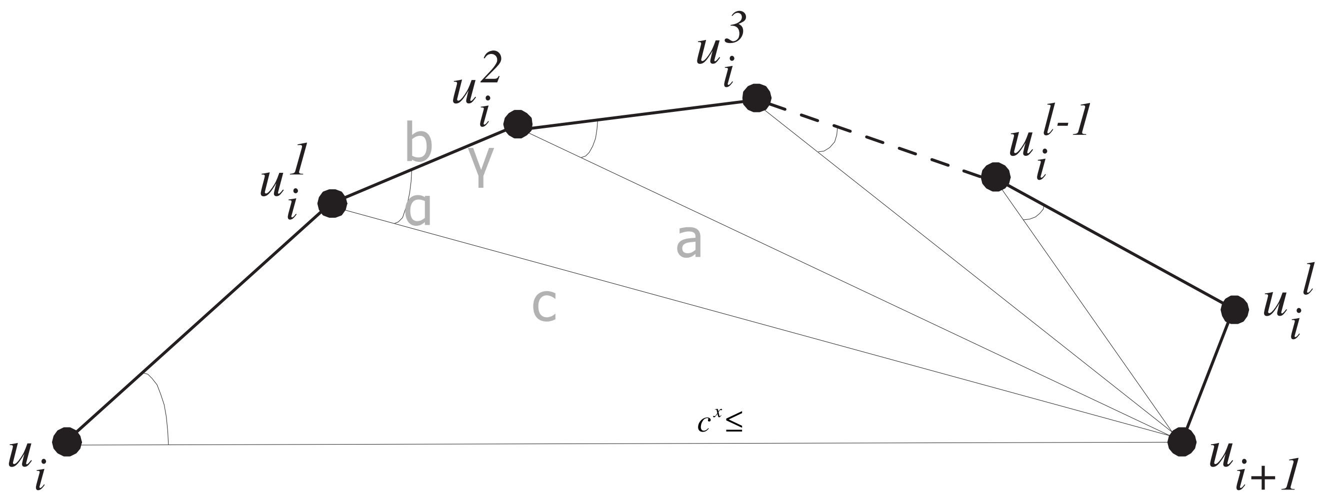
Case 1: $d(u_i, u^1_i) < d(u^1_i, u_{i+1})$



Case 2: $d(u_i, u^1_i) \geq d(u^1_i, u_{i+1})$



Constructive Proof



For r*=(s=u_1,u_2,…,u_k=t) in G', construct a path r in G by finding a path from u_i to u_{i+1} .

By algorithm construction, exists an next node such that α≤4/π,

Case 1: if a>b, then γ>π/2, and a<c, and therefore $a^x + b^x \le c^x$

Case 2: if a<b, done, and $a^x + b^x \le c^x(1+2\sin(\alpha/2))$

Combined with power model to prove the power spanner property.

Summary

- Local interactions result in global properties.

- Topology control for other properties
  - E.g., greedy forwarding.