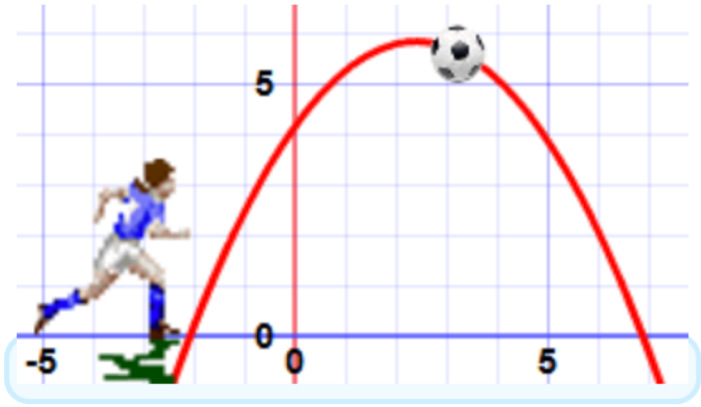
Quadratic Equation

Dengan metode Completing The Square

Quadratic Eq

this makes it Quadratic

$5x^2 + 3x + 3 = 0$



Quadratic Eq

Standard Form

The **Standard Form** of a Quadratic Equation looks like this:

$$ax^2 + bx + c = 0$$

- **a**, **b** and **c** are known values. **a** can't be 0.
- "**x**" is the **variable** or unknown (we don't know it yet).

Balls, Arrows, Missiles and Stones

When you throw a ball (or shoot an arrow, fire a missile or throw a stone) it goes up into the air, slowing as it travels, then comes down again faster and faster ...

... and a Quadratic Equation tells you its position at all times!

Example: Throwing a Ball

With conditions :

The height starts at 3 m:		3
It travels upwards at 14 meters per second (14 m/s):		14t
Gravity pulls it down, changing its position by *about* 5 m per second squared:		$-5t^2$
(Note for the enthusiastic: the $-5t^2$ is simplified from $-(½)at^2$ with $a=9.8\ m/s^2$)		

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Add them up and the height **h** at any time **t** is:

$$h = 3 + 14t - 5t^2$$

And the ball will hit the ground when the height is zero:

$$3 + 14t - 5t^2 = 0$$

In "Standard Form" it looks like:

$$-5t^2 + 14t + 3 = 0$$

Factorization :

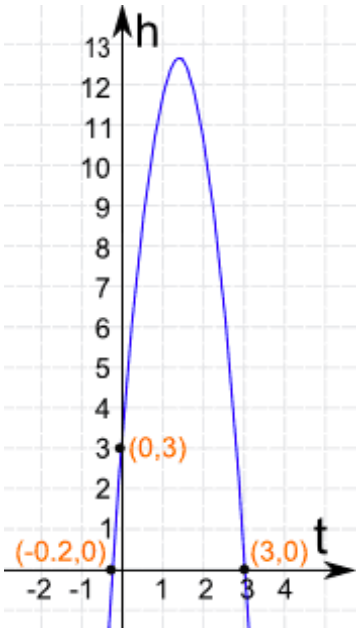
Rewrite middle with −15 and 1: $5t^2 - 15t + t - 3 = 0$

Factor first two and last two: $5t(t - 3) + 1(t - 3) = 0$

Common Factor is (t − 3): $(5t + 1)(t - 3) = 0$

And the two solutions are: $5t + 1 = 0$ or $t - 3 = 0$

$t = \mathbf{-0.2}$ or $t = \mathbf{3}$



The ball hits the ground after 3 seconds!

Here is the graph of the Parabola $h = -5t^2 + 14t + 3$

It shows you the **height** of the ball vs **time**

Some interesting points:

(0,3) When t=0 (at the start) the ball is at 3 m

(−0.2,0) says that −0.2 seconds BEFORE we threw the ball it was at ground level. This never happened! So our common sense says to ignore it.

(3,0) says that at 3 seconds the ball is at ground level.

Also notice that the ball goes **nearly 13 meters** high.

Formula Quadratic

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The ± means there are TWO answers:

$$x = \frac{-b + \sqrt{(b^2 - 4ac)}}{2a}$$

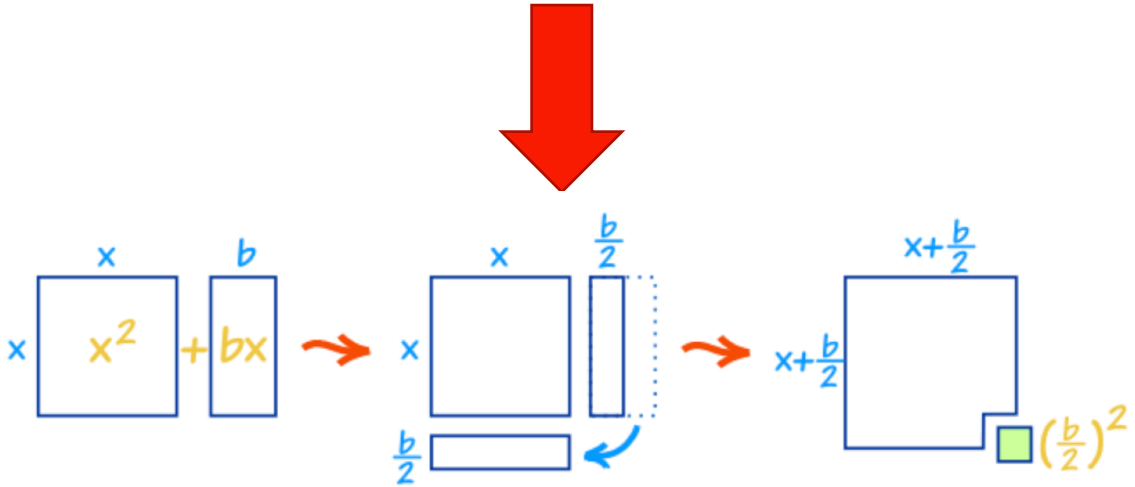
$$x = \frac{-b - \sqrt{(b^2 - 4ac)}}{2a}$$

Completing The Square Method!

… take a Quadratic Equation like this:		and turn it into this:
$ax^2 + bx + c = 0$		$a(x+d)^2 + e = 0$

Completing The Square Method!

Misalkan ambil contoh : $x^2 + bx$



$x^2 + bx \quad + (b/2)^2 \quad = \quad (x+b/2)^2$

Completing The Square Method!

Start with: $x^2 + 6x + 7$

("b" is 6 in this case)

Complete the Square:

$$x^2 + 6x + \square + 7 \square$$

$$+\left(\frac{6}{2}\right)^2 \qquad -\left(\frac{6}{2}\right)^2$$

Also **subtract** the new term

Simplify it and we are done.

$$\underbrace{x^2 + 6x + \left(\frac{6}{2}\right)^2}_{\left(x+\frac{6}{2}\right)^2} + \underbrace{7 - \left(\frac{6}{2}\right)^2}_{7-9} = (x+3)^2 - 2$$

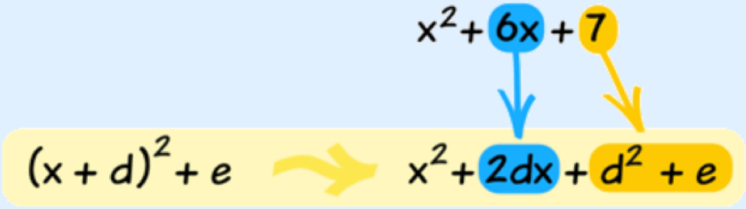
The result:

$$x^2 + 6x + 7 = (x+3)^2 - 2$$

And now x only appears once, and our job is done!

Completing The Square Method!

Example: try to fit $x^2 + 6x + 7$ into $x^2 + 2dx + d^2 + e$



Now we can "force" an answer:

- We know that $6x$ must end up as $2dx$, so **d must be 3**
- Next we see that 7 must become $d^2 + e = 9 + e$, so **e must be −2**

And we get the same result $(x+3)^2 - 2$ as above!

Example 1: Solve $x^2 + 4x + 1 = 0$

Step 1 can be skipped in this example since the coefficient of x^2 is 1

Step 2 Move the number term to the right side of the equation:

$\rightarrow x^2 + 4x = -1$

Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation.

$(b/2)^2 = (4/2)^2 = 2^2 = 4$

$\rightarrow x^2 + 4x + 4 = -1 + 4$

$\rightarrow (x + 2)^2 = 3$

Step 4 Take the square root on both sides of the equation:

$\rightarrow x + 2 = \pm\sqrt{3} = \pm 1.73$ (to 2 decimals)

Step 5 Subtract 2 from both sides:

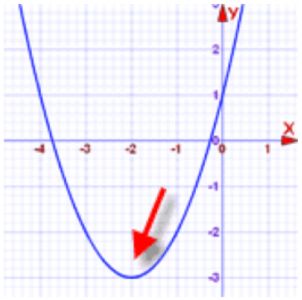
$\rightarrow x = \pm 1.73 - 2 = -3.73$ or -0.27

And here is an interesting and useful thing.

At the end of step 3 we had the equation:

$\rightarrow (x + 2)^2 = 3$

It gives us the **vertex** (turning point) of $x^2 + 4x + 1$: **(-2, -3)**



Example 2: Solve $5x^2 - 4x - 2 = 0$

Step 1 Divide all terms by 5

$x^2 - 0.8x - 0.4 = 0$

Step 2 Move the number term to the right side of the equation:

$x^2 - 0.8x = 0.4$

Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation:

$(b/2)^2 = (0.8/2)^2 = 0.4^2 = 0.16$

$x^2 - 0.8x + 0.16 = 0.4 + 0.16$

$(x - 0.4)^2 = 0.56$

Step 4 Take the square root on both sides of the equation:

$x - 0.4 = \pm\sqrt{0.56} = \pm 0.748$ (to 3 decimals)

Step 5 Subtract (-0.4) from both sides (in other words, add 0.4):

$x = \pm 0.748 + 0.4 = -0.348$ or 1.148

Keuntungan

- ✓ Dapat digunakan untuk mengetahui vertex (titik balik)
- ✓ Persamaan jadi lebih mudah di selesaikan
- ✓ Formula kudratik diturunkan dari metode completing squared

Formula kudratik

Start with	$ax^2 + bx + c = 0$
Divide the equation by a	$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
Put c/a on other side	$x^2 + \frac{b}{a}x = -\frac{c}{a}$
Add $(b/2a)^2$ to both sides	$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
*The **left hand side** is now in the **$x^2 + 2dx + d^2$** format, where "d" is "b/2a" So we can re-write it this way:*	
"Complete the Square"	$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

Formula kudratik

2. Now Solve For "x"

Now we just need to rearrange the equation to leave "x" on the left

Start with $$\left(x+\frac{b}{2a}\right)^2=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2$$

Square root $$x+\frac{b}{2a}=\pm\sqrt{-\frac{c}{a}+\left(\frac{b}{2a}\right)^2}$$

Move b/2a to right $$x=-\frac{b}{2a}\pm\sqrt{-\frac{c}{a}+\left(\frac{b}{2a}\right)^2}$$

That is actually solved! But let's simplify it a bit:

Multiply right by 2a/2a $$x=\frac{-b\pm\sqrt{-\frac{c}{a}\times(2a)^2+\left(\frac{b}{2a}\right)^2\times(2a)^2}}{2a}$$

Simplify: $$x=\frac{-b\pm\sqrt{-4ac+b^2}}{2a}$$

Which is the Quadratic formula we all know and love:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$